PSU CS 251 Midterm 1 Spring 2010 **Exam Key**

Before you start, ***write your name at the top of each page****. This exam was designed to take you 30 minutes or so; you will be given 60. The exam is open-book open-notes. Enough space should be given for each solution, but if not then indicate this and continue on the back.*

I suggest that you read the entire exam before you start. If you find a problem with the exam, please note it in your answer and answer as best you can. Please show as much of your work as you reasonably can: I cannot give partial credit for your invisible work.

Some questions are marked choose one*. Others are marked* choose all that apply*. Make sure that you understand which kind you are answering before you answer. On* choose all that apply *questions, it might be that none of the choices apply, or that all of them do—any subset of the choices is a valid possibility.*

1. Indicate whether each of the following formulae is a tautology (t), is a contingency (c), or is a contradiction (u for unsatisfiable) by circling the appropriate letter. When the formula is a contingency, give an interpretation that makes it true, and an interpretation that makes it false. *Circle one of* t*,* c*, or* u*.*

   (a) *[4 pts] (T) c u :* $A \vee B \vee \neg C \vee \neg A$

   ☐

   (b) *[4 pts] t c (U) :* $A \wedge B \wedge C \wedge (B \rightarrow \neg A)$

   ☐

   (c) *[4 pts] t c (U) :* $\neg A \wedge \neg(A \vee (A \rightarrow A))$

   ☐

   (d) *[4 pts] t (C) u :* $A \rightarrow \neg A$

   A=false; A=true

   (e) *[4 pts] t (C) u :* $(A \vee B) \rightarrow A$

   A=true, B=false; A=false, B=true

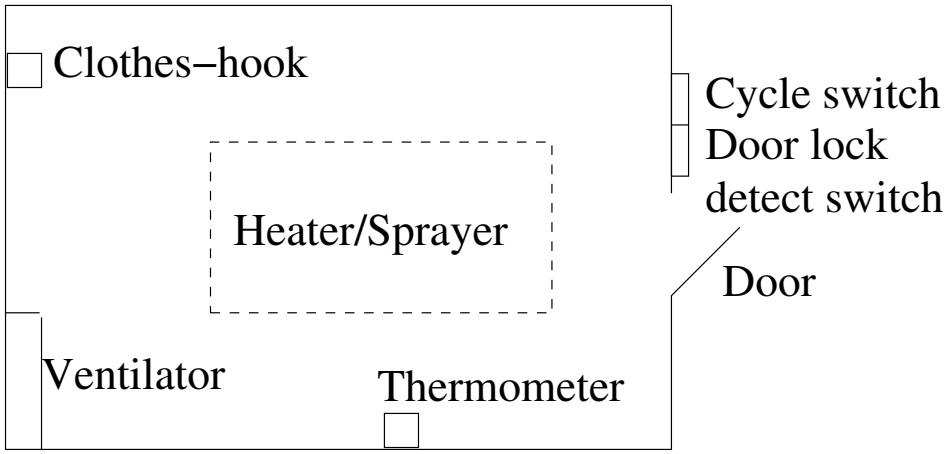

Figure 1: Paint-and-Bake Room, Top View

2. An industrial spray-and-bake room at a painting company has a number of safeguards to keep employees from injury, while trying to maximize the use of the room.

   The room contains two automatically-controlled devices: a paint sprayer with a high-temperature heater that actually do the paint-and-bake (D_C, cycle is running when true) and a ventilator (D_V, on when true). It also contains four sensors: a clothes-hook with a switch indicating whether a thermal-protection suit is hanging from it (S_P, true when hook occupied—it is assumed that when the hook is unoccupied the worker is wearing the suit), a door-locked detection switch (S_L, true when locked—the door can be locked only from the outside), a thermometer calibrated to indicate whether the room is too hot to be safe for an unprotected person (S_T, true when too hot to be inside the room but outside the suit), a switch to operate the room (S_C, true when the room is supposed to be running).

   For each of the device outputs D, write a propositional formula in terms of the sensor inputs S that describes their operation. The formula should describe when the device output should be true, satisfying the following safety and performance conditions:

   - The heater/spray cycle should be running when the operation switch is on. However, the cycle should not run when the door is unlocked; neither should it run when the clothes-hook is occupied and the temperature is too high.
   - The ventilator should be on when the operation switch is on. It should also be on whenever the door is unlocked, the clothes-hook is occupied, and the temperature is too high.

   D_C should be true as often as possible given these conditions, so that machine time is not wasted. D_V should be false as often as possible, to minimize use of the ventilator.

(a) *[25 pts]* Write a formula for D_C.

$D_C \equiv S_C \wedge S_L \wedge \neg(S_P \wedge S_T)$

(b) *[25 pts]* Write a formula for D_V.

$D_V \equiv S_C \vee (\neg S_L \wedge S_P \wedge S_T)$

(c) *[30 pts]* Given your formula for D_V above, show that when the ventilator is on, if the cycle switch is off then the temperature is too high. Use a deductive proof.

$$D_V \rightarrow (S_C \rightarrow S_L)$$

1.	D_V	[P]
2.	$S_C \vee (\neg S_L \wedge S_P \wedge \neg S_T)$	[1, def. D_V]
3.	$\neg S_C \rightarrow (\neg S_L \wedge S_P \wedge \neg S_T)$	[2, def. $\rightarrow$]
4.	$\quad \neg S_C$	[P]
5.	$\quad \neg S_L \wedge S_P \wedge S_T$	[4, 3, MP]
6.	$\quad S_T$	[5, Simp]
7.	$\neg S_C \rightarrow S_T$	[4–6, CP]
$\therefore$		[1–7, CP]