Prop. Logic for Traffic Signals

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Consider traffic signals. Specifically, consider the traffic signals at this intersection.

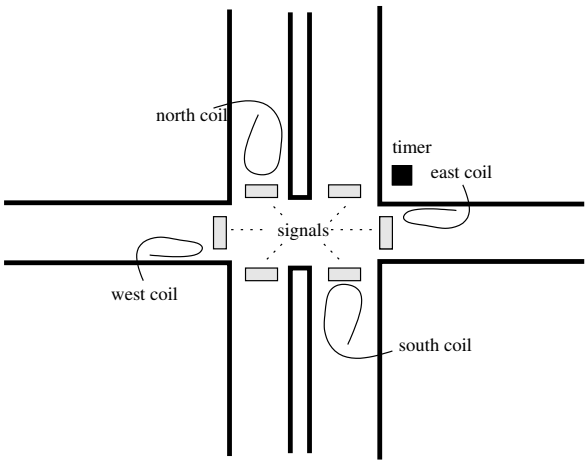

Figure 1: Intersection

The intersection has signal lights that control cars traveling north-south and east-west. Each of those signals lights can be either red or green (we assume the yellow is handled automatically and out of scope). We will denote the states of the signals using the atoms S_{NR}, S_{NG}, S_{ER} and S_{EG} (for "north red", "north green", "east red" and "east green"). The intersection also has coils embedded in the north, south, east and west pavement that activate when a car is there. Call these C_N, C_S, C_E, and C_W. Finally, there is a timer called T that is true when North-South traffic should have priority, and false when East-West traffic should (under the circumstance that there is waiting traffic in both directions).

The intersection is governed by the following rules:

- When the intersection is empty, or when traffic is waiting only in the north-south direction, the signal should be green for north-south traffic and red for east-west.
- When there is no waiting north-south traffic but is waiting east-west traffic, the north-south signal should be red and the east-west signal should be green.
- When there is waiting traffic in both directions, the direction of green should be controlled by the timer.

1. Write equations for S_{NR}, S_{NG}, S_{ER} and S_{EG} in terms of the other variables.
2. Verify the following properties of your equations using deductive proof:
   (a) Neither the east-west nor the north-south signal ever is both green and red simultaneously.
   (b) The east-west and north-south signals are never both green simultaneously.
   (c) At any time, one of the signals is green.
   (d) When the intersection is empty, the east-west signal is red.