PSU CS 251 Midterm 2 Spring 2010 **Exam Key**

Before you start, ***write your name at the top of each page****. This is a take-home exam. It was designed to take you 45 minutes or so; you are required to stop after two hours. Please take the exam in at most two sittings; i.e., one break only.*

The exam is open-book open-notes (where "notes" means notes you've taken in advance in class or as you read the text). It is not open anything else. In particular, use of the Internet or friends is cheating.

Enough space should be given for each solution, but if not then indicate this and continue on the back. I suggest that you read the entire exam before you start. If you find a problem with the exam, please note it in your answer and answer as best you can. Please show as much of your work as you reasonably can: I cannot give partial credit for your invisible work.

1. For each of the following indicate that (a) it is not a well-formed first-order formula, (b) that it is invalid, (c) that it is not valid but is satisfiable in some interpretation, or (d) that it is valid. Assume that all predicates and functions have ranges and domains in the natural numbers, and that $s = 7$ in every interpretation. *(Circle a, b, c, or d)*

   (a) *[6 pts] a b (C) d :* $\forall x\ \exists y\ p(x, y) \equiv \exists y\ \forall x\ p(x, y)$

   (b) *[6 pts] a b c (D) :* $p(7) \wedge p(s) \equiv p(s)$

   (c) *[6 pts] (A) b c d :* $\forall x\ x \vee \neg x$

   (d) *[6 pts] a b c (D) :* $p(s) \wedge \forall x\ \neg q(x) \equiv \neg\exists x\ (\neg p(s) \vee q(x))$

   (e) *[6 pts] a b c (D) :* $\exists x\ \exists y\ p(x, y) \equiv \exists y\ \exists x\ p(x, y) \equiv \exists y\ \exists x\ p(y, x)$

   (f) *[6 pts] a (B) c d :* $\forall y\ \exists x\ y = x \cdot x$

2. *[24 pts]* Try to unify the following predicate instances, using any method that you like. Show your work. Either indicate that the unification fails, or give a substitution set for an MGU.

$$p(x, f(x, y), g(f(z))) \quad p(f(z), f(f(z), z), g(x))$$

$$\theta = \{x/f(z), y/z\}$$

Alternatively, you could argue, for full credit, that the problem is ill-defined or fails since f appears with two different arities—my mistake.

3. *[40 pts]* Prove or disprove: there is no largest even natural number. (An even natural number is any number that you can get by multiplying some other natural number by two.)

$$\neg\exists w\ (\exists x\ w = 2 \cdot x \wedge \forall y\ (\exists z\ y = 2 \cdot z \rightarrow w \geq y))$$

1.	$\exists w\ (\exists x\ w = 2 \cdot x \wedge \forall y\ (\exists z\ y = 2 \cdot z \rightarrow w \geq y))$	[P (for IP)]
2.	$\exists x\ k_w = 2 \cdot x \wedge \forall y\ (\exists z\ y = 2 \cdot z \rightarrow k_w \geq y)$	[1, EI on w]
3.	$\forall y\ (\exists z\ y = 2 \cdot z \rightarrow k_w \geq y)$	[2, Simp]
4.	$\exists z\ 2 \cdot (k_w + 1) = 2 \cdot z \rightarrow k_w \geq 2 \cdot (k_w + 1)$	[3, UI on y]
5.	*true* $\rightarrow$ *false*	[4, T]
6.	*false*	[5, def $\rightarrow$]
$\therefore$		[1–6, IP]