

Data Representation

321

(in decimal)

100 10 1

← How did we get these?

↑ ↑ ↑
 10^2 10^1 10^0

← “Base-10”

↑ ↑ ↑
 $3 \cdot 100 + 2 \cdot 10 + 1 \cdot 1$

← Positions denote powers of 10
Digits 0-9 denote position values

10100001

101 million and one?

Actually, 321 in binary (Base-2)

Why should we care?

- Computers use binary (bits) to store all information

101000001

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑
256 128 64 32 16 8 4 2 1

← How did we
get these?

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑
 2^8 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

← “Base-2”

$$256 + 0 + 64 + 0 + 0 + 0 + 0 + 0 + 1 = 321$$



Positions denote powers of 2
Digits 0 and 1 denote position values

Binary group activity

Find decimal values of binary numbers below

$$\begin{array}{r} 16 \ 8 \ 4 \ 2 \ 1 \\ x = 10011 \\ y = 01001 \end{array}$$

What are the decimal values of

$$x-y \quad x+y$$

What is the binary representation of

$$x-y \quad x+y$$

0x141

141 in decimal?

Actually, 321 in hexadecimal (Base-16)

Why hexadecimal?

Recall binary for 321

000101000001

0001 0100 0001
↓ ↓ ↓
1 4 1

0x141



256 16 1 ← How did we get these?



16^2 16^1 16^0 ← “Base-16”



$$1*256 + 4*16 + 1*1 = 321$$



Positions denote powers of 16

But, requires 16 digit settings to denote values

- Binary 0,1
- Decimal 0,1,2,3,4,5,6,7,8,9
- Hexadecimal 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

| Hex | Decimal | Binary |
|-----|---------|--------|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

Hexadecimal example

0x0CD



256 16 1



$$0*256 + 12*16 + 13*1$$

$$192 + 13 = 205$$

| Hex | Decimal | Binary |
|-----|---------|--------|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

Hexadecimal activity

0x1AF

↑ ↑ ↑

256 16 1

↑ ↑ ↑

$$1 * 256 + 10 * 16 + 15 * 1$$

$$256 + 160 + 15 = 431$$

| Hex | Decimal | Binary |
|-----|---------|--------|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

Converting bases

From Base 10 to other bases

- Find largest power x of base less than number n
- Find largest base digit b where $b*x < n$
- Recursively repeat using $n - (b*x)$

Example

- $15213_{10} = 1*10^4 + 5*10^3 + 2*10^2 + 1*10^1 + 3*10^0$

Base 2 (binary)

| | | | | | | | | | | | | | | | | |
|--|-------|------|------|------|------|-----|-----|-----|----|----|----|---|---|---|---|----------------|
| $x=2$ | 16384 | 8192 | 4096 | 2048 | 1024 | 512 | 256 | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | |
| 15213_{10} | = | 0 | 1 | | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 ₂ |
| $11101101101101_2 = 1*2^{13} + 1*2^{12} + 1*2^{11} + 0*2^{10} + 1*2^9 + \text{etc...}$ | | | | | | | | | | | | | | | | |

Converting bases

Example

- $15213_{10} = 1*10^4 + 5*10^3 + 2*10^2 + 1*10^1 + 3*10^0$

Base 16 (hexadecimal)

- » $x=16 \quad 65536 \quad 4096 \quad 256 \quad 16 \quad 1$

- » $15213_{10} = 0 \quad 3 \quad B \quad 6 \quad D_{16}$

- » $3B6D_{16} = 3*16^3 + 11*16^2 + 6*16^1 + 13*16^0$

- » Written in C as 0x3b6d

From Base 2 binary to bases that are powers of two

- Done by grouping bits and assigning digits

Base 2 (binary)

- » $x=2 \quad 16384 \quad 8192 \quad 4096 \quad 2048 \quad 1024 \quad 512 \quad 256 \quad 128 \quad 64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$

- » $15213_{10} = 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1_2$

- » $11101101101101_2 = 1*2^{13} + 1*2^{12} + 1*2^{11} + 0*2^{10} + 1*2^9 + \text{etc...}$

- Example of binary to hex

- $11 \ 1011 \ 0110 \ 1101 = 3 \ B \ 6 \ D$

Practice

Convert the following

- 10110111_2 to Base 10
- 11011001_2 to Base 16
- $0x2ae$ to Base 2
- $0x13e$ to Base 10
- 150_{10} to Base 2
- 301_{10} to Base 16

Base 2

128 64 32 16 8 4 2 1

Base 16

256 16 1

Hex digits

a=10= 1010_2
b=11= 1011_2
c=12= 1100_2
d=13= 1101_2
e=14= 1110_2
f=15= 1111_2

0x333231

3,355,185?

Actually, “321” in ASCII

Humans encode characters in pairs of hexadecimal digits

- Each pair of hex digits is 8 bits or 1 byte
- Bytes are the smallest unit of data for computers

0x333231

```

    0x33      0x32      0x31
    '3'       '2'       '1'
  
```

ASCII maps byte values to characters

- Used in URLs, web pages, e-mail
- Other representations (Unicode, EBCDIC)

ASCII TABLE

| Decimal | Hex | Char | Decimal | Hex | Char | Decimal | Hex | Char | Decimal | Hex | Char |
|---------|-----|------------------------|---------|-----|---------|---------|-----|------|---------|-----|-------|
| 0 | 0 | [NULL] | 32 | 20 | [SPACE] | 64 | 40 | @ | 96 | 60 | ` |
| 1 | 1 | [START OF HEADING] | 33 | 21 | ! | 65 | 41 | A | 97 | 61 | a |
| 2 | 2 | [START OF TEXT] | 34 | 22 | " | 66 | 42 | B | 98 | 62 | b |
| 3 | 3 | [END OF TEXT] | 35 | 23 | # | 67 | 43 | C | 99 | 63 | c |
| 4 | 4 | [END OF TRANSMISSION] | 36 | 24 | \$ | 68 | 44 | D | 100 | 64 | d |
| 5 | 5 | [ENQUIRY] | 37 | 25 | % | 69 | 45 | E | 101 | 65 | e |
| 6 | 6 | [ACKNOWLEDGE] | 38 | 26 | & | 70 | 46 | F | 102 | 66 | f |
| 7 | 7 | [BELL] | 39 | 27 | ' | 71 | 47 | G | 103 | 67 | g |
| 8 | 8 | [BACKSPACE] | 40 | 28 | (| 72 | 48 | H | 104 | 68 | h |
| 9 | 9 | [HORIZONTAL TAB] | 41 | 29 |) | 73 | 49 | I | 105 | 69 | i |
| 10 | A | [LINE FEED] | 42 | 2A | * | 74 | 4A | J | 106 | 6A | j |
| 11 | B | [VERTICAL TAB] | 43 | 2B | + | 75 | 4B | K | 107 | 6B | k |
| 12 | C | [FORM FEED] | 44 | 2C | , | 76 | 4C | L | 108 | 6C | l |
| 13 | D | [CARRIAGE RETURN] | 45 | 2D | - | 77 | 4D | M | 109 | 6D | m |
| 14 | E | [SHIFT OUT] | 46 | 2E | . | 78 | 4E | N | 110 | 6E | n |
| 15 | F | [SHIFT IN] | 47 | 2F | / | 79 | 4F | O | 111 | 6F | o |
| 16 | 10 | [DATA LINK ESCAPE] | 48 | 30 | 0 | 80 | 50 | P | 112 | 70 | p |
| 17 | 11 | [DEVICE CONTROL 1] | 49 | 31 | 1 | 81 | 51 | Q | 113 | 71 | q |
| 18 | 12 | [DEVICE CONTROL 2] | 50 | 32 | 2 | 82 | 52 | R | 114 | 72 | r |
| 19 | 13 | [DEVICE CONTROL 3] | 51 | 33 | 3 | 83 | 53 | S | 115 | 73 | s |
| 20 | 14 | [DEVICE CONTROL 4] | 52 | 34 | 4 | 84 | 54 | T | 116 | 74 | t |
| 21 | 15 | [NEGATIVE ACKNOWLEDGE] | 53 | 35 | 5 | 85 | 55 | U | 117 | 75 | u |
| 22 | 16 | [SYNCHRONOUS IDLE] | 54 | 36 | 6 | 86 | 56 | V | 118 | 76 | v |
| 23 | 17 | [ENG OF TRANS. BLOCK] | 55 | 37 | 7 | 87 | 57 | W | 119 | 77 | w |
| 24 | 18 | [CANCEL] | 56 | 38 | 8 | 88 | 58 | X | 120 | 78 | x |
| 25 | 19 | [END OF MEDIUM] | 57 | 39 | 9 | 89 | 59 | Y | 121 | 79 | y |
| 26 | 1A | [SUBSTITUTE] | 58 | 3A | : | 90 | 5A | Z | 122 | 7A | z |
| 27 | 1B | [ESCAPE] | 59 | 3B | ; | 91 | 5B | [| 123 | 7B | { |
| 28 | 1C | [FILE SEPARATOR] | 60 | 3C | < | 92 | 5C | \ | 124 | 7C | |
| 29 | 1D | [GROUP SEPARATOR] | 61 | 3D | = | 93 | 5D |] | 125 | 7D | } |
| 30 | 1E | [RECORD SEPARATOR] | 62 | 3E | > | 94 | 5E | ^ | 126 | 7E | ~ |
| 31 | 1F | [UNIT SEPARATOR] | 63 | 3F | ? | 95 | 5F | _ | 127 | 7F | [DEL] |

ASCII activity

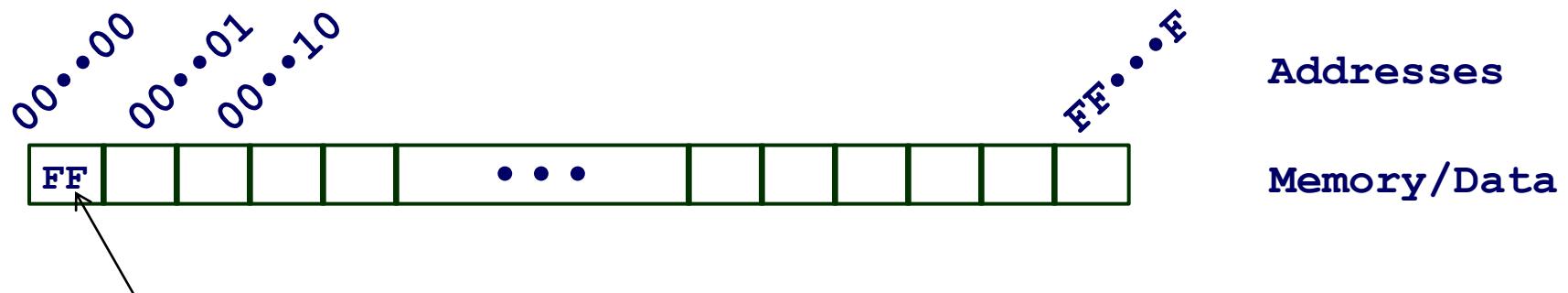
Snippet #1: 54 68 65 20 68 6F 6D 65 20 74 6F
Snippet #2: 65 76 65 72 79 6F 6E 65 20 69 73
Snippet #3: 74 6F 20 68 69 6D 20 68 69 73
Snippet #4: 63 61 73 74 6C 65 20 61 6E 64
Snippet #5: 66 6F 72 74 72 65 73 73 2E
Snippet #6: 45 64 77 61 72 64 20 43 6F 6B 65

ASCII TABLE

| Decimal | Hex | Char | Decimal | Hex | Char | Decimal | Hex | Char | Decimal | Hex | Char |
|---------|-----|------------------------|---------|-----|---------|---------|-----|------|---------|-----|-------|
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| 21 | 15 | [NEGATIVE ACKNOWLEDGE] | 53 | 35 | 5 | 85 | 55 | U | 117 | 75 | u |
| 22 | 16 | [SYNCHRONOUS IDLE] | 54 | 36 | 6 | 86 | 56 | V | 118 | 76 | v |
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| 24 | 18 | [CANCEL] | 56 | 38 | 8 | 88 | 58 | X | 120 | 78 | x |
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Byte-Oriented Memory Organization

Memory organized as an array of bytes



- **Byte = 8 bits**
 - Binary 00000000_2 to 11111111_2
 - Decimal: 0_{10} to 255_{10}
 - Hexadecimal 00_{16} to FF_{16}
- **Address is an index into array**
- **Addressable unit of memory is a byte**
- **Recall system provides private address spaces to each “process”**

Machine Words

Any given computer has a “Word Size”

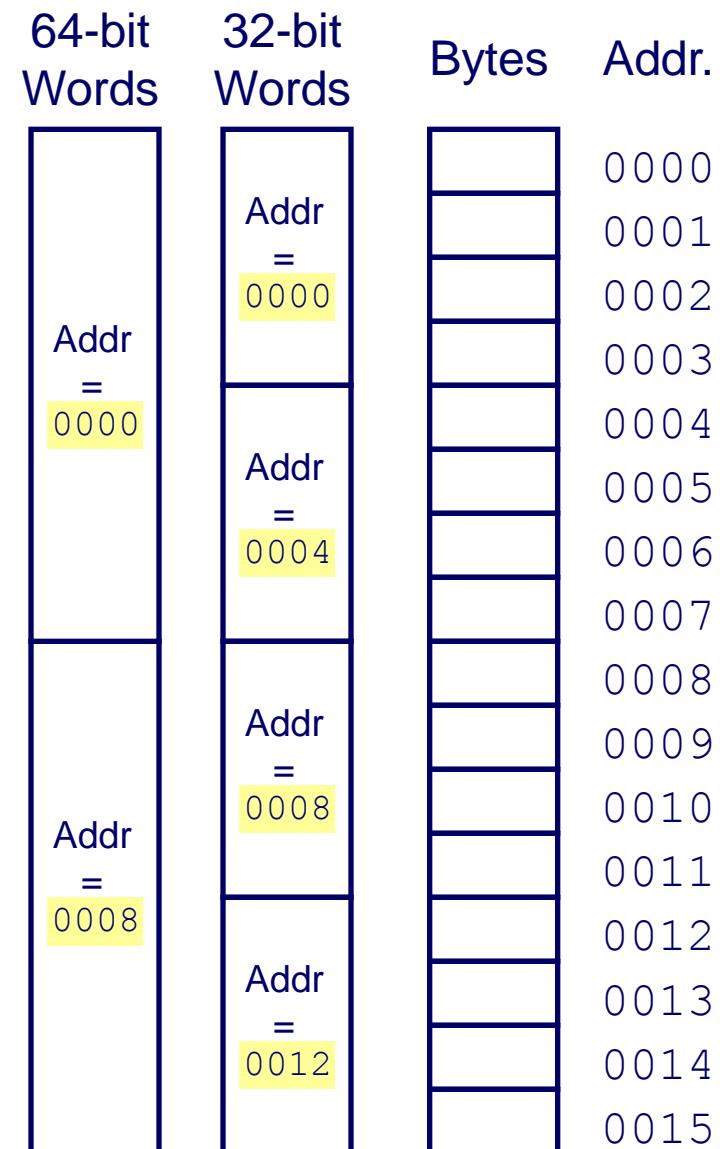
- Nominal size of integer-valued data
- Until recently, x86 word size was 32 bits (4 bytes)
 - Limits addresses to 4GB (2^{32} bytes)
- Now 64-bit word size
 - Potentially up to 18 PB (petabytes) of addressable memory
 - That's 18.4×10^{15} bytes of addressable memory

Word-Oriented Memory Organization

Addresses Specify Byte Locations

Address of first byte in word

Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Byte Ordering

How should bytes within multi-byte word be ordered in memory?

Conventions

- Sun, PowerPC Macs, Internet protocols are “BigEndian”
 - Least significant byte has highest address
- x86 (PC/Mac), ARM (Android/iOS) are “LittleEndian”
 - Least significant byte has lowest address

Byte Ordering Example

Big Endian

- Least significant byte has highest address

Little Endian

- Least significant byte has lowest address (LLL)

Example

- Variable **x** has 4-byte representation **0x01234567**
- Address given by **&x** is **0x100**

Big Endian

0x100 0x101 0x102 0x103



Little Endian

0x100 0x101 0x102 0x103



Endian

How do you test endian-ness?

- Direct inspection of memory via gdb
 - endian

- » **gdb endian**
- » **break 5**
- » **run**
- » **p /x &i**
- » **x/b &i**
- » **x/b ...**

```
#include <stdio.h>
main()
{
    int i=0x01020304;
    printf("%d\n",i);
}
```

<http://thefengs.com/wuchang/courses/cs201/class/04/endian.c>

Endian

How do you test endian-ness?

- Simple program from book (`show_bytes`)

```
#include <stdio.h>
#include <string.h>
typedef unsigned char *byte_pointer;
void show_bytes(byte_pointer start, int len)
{
    int i;
    for (i = 0; i < len; i++)
        printf(" %.2x", start[i]);
    printf("\n");
}

void show_int(int x)
{    show_bytes((byte_pointer) &x, sizeof(int));}

void show_float(float x)
{    show_bytes((byte_pointer) &x, sizeof(float));}

void show_pointer(void *x)
{    show_bytes((byte_pointer) &x, sizeof(void*));}

int main()
{
    int i=0x01020304;
    float f=2345.6;
    int *ip=&i;
    char *s = "ABCDEF";

    show_int(i);
    show_float(f);
    show_pointer(ip);
    show_bytes(s,strlen(s));
}
```

Output:

```
04 03 02 01
9a 99 12 45
28 61 61 63 fc 7f 00 00
41 42 43 44 45 46
```

Representing pointers

Recall

A **pointer** is a variable containing a memory address of an object of a particular data type

- Contains a “reference” address for data

```
char* cp; /* Declares cp to be a pointer to a character */  
int* ip; /* Declares ip to be a pointer to an integer */
```

- How many bytes is cp?
- How many bytes is ip?
- Both store address locations

Pointers in memory

Given the following code on x86-64...

```
main()
{
    int B = -15213;
    int* P = &B;
}
```

x86-64

| |
|----|
| D8 |
| F8 |
| FF |
| 07 |
| 00 |
| 00 |
| 00 |
| 00 |

Suppose the address of B is 0x007ffff8d8
and the address of P is 0x007ffff8d0

What is the size of P?

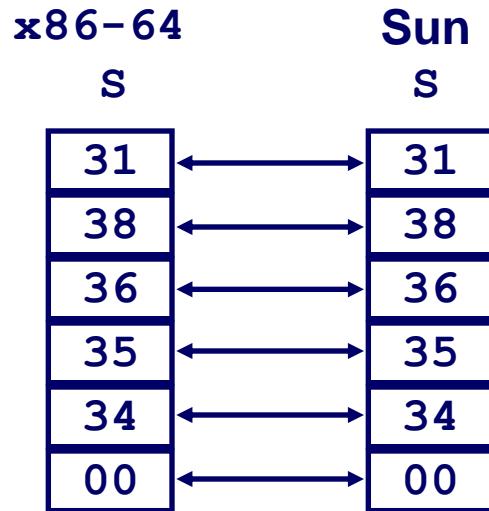
At the end of main, write the value of each byte
of P in order as it appears in memory.

Representing strings

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
 - Standard 7-bit encoding of character set
 - Character “0” has code 0x30
 - » Digit i has code $0x30+i$
- Must be null-terminated
 - Final character = 0

```
char S[6] = "18654";
```



Compatibility

- Endian is not an issue
 - Data are single byte quantities
- Text files generally platform independent
 - Except for different conventions of line termination character(s)!

Representing strings

Alternate Unicode encoding

- 7-bit ASCII only suitable for English text
 - Can not cover characters in all languages
- 16-bit unicode character set
 - Supports Greek, Russian, Chinese, etc.
- Default encoding for strings in Java
- Support in C libraries for character set

Representing integers (signed)

Support for two types

- unsigned and signed
- Both are the same size (4 bytes or 32-bits on IA32)
- Differ based on how bits are interpreted

Unsigned integers

```
unsigned int i;  
printf("%u\n", i)
```

- Encodes 0 to $(2^{32} - 1)$
- 0 to 4294967295
- Exactly as described in binary number slides

Signed integers in 2's complement format (default)

```
int i;  
printf("%d\n", i)
```

- Encodes -2^{31} to $(2^{31}-1)$
- -2,147,483,648 to 2,147,483,647

Encoding Integers

Unsigned

$$\begin{aligned}B2U(X) &= \sum_{i=0}^{w-1} x_i \cdot 2^i \\&= x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i\end{aligned}$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

MSB
if 1, negative



16-bit example

■ Unsigned

32768 16384 8192 4096 2048 1024 512 256 128 64 32 16 8 4 2 1

■ Signed

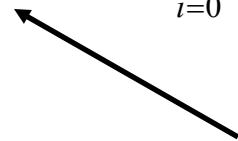
-32768 16384 8192 4096 2048 1024 512 256 128 64 32 16 8 4 2 1

Encoding Integers

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
short int x = 15213;  
short int y = -15213;
```



MSB
if 1, negative

C short 2 bytes long

| | Decimal | Hex | Binary |
|----------|---------|-------|-------------------|
| x | 15213 | 3B 6D | 00111011 01101101 |
| y | -15213 | C4 93 | 11000100 10010011 |

Two-complement Encoding Example (Cont.)

| | | | |
|-----|---------|----------|----------|
| x = | 15213: | 00111011 | 01101101 |
| y = | -15213: | 11000100 | 10010011 |

| Weight | 15213 | | -15213 | |
|--------|-------|-------|--------|--------|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 2 |
| 4 | 1 | 4 | 0 | 0 |
| 8 | 1 | 8 | 0 | 0 |
| 16 | 0 | 0 | 1 | 16 |
| 32 | 1 | 32 | 0 | 0 |
| 64 | 1 | 64 | 0 | 0 |
| 128 | 0 | 0 | 1 | 128 |
| 256 | 1 | 256 | 0 | 0 |
| 512 | 1 | 512 | 0 | 0 |
| 1024 | 0 | 0 | 1 | 1024 |
| 2048 | 1 | 2048 | 0 | 0 |
| 4096 | 1 | 4096 | 0 | 0 |
| 8192 | 1 | 8192 | 0 | 0 |
| 16384 | 0 | 0 | 1 | 16384 |
| -32768 | 0 | 0 | 1 | -32768 |
| Sum | | 15213 | -15213 | |

Two's complement exercise

Exercise: Write -3 , -4, and –5 in two's complement format for w=4

| | | | |
|-------|---|---|---|
| -8 | 4 | 2 | 1 |
| <hr/> | | | |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |

| X | B2U(X) | B2T(X) |
|------|------------|------------|
| 0000 | 0 | 0 |
| 0001 | 1 | 1 |
| 0010 | 2 | 2 |
| 0011 | 3 | 3 |
| 0100 | 4 | 4 |
| 0101 | 5 | 5 |
| 0110 | 6 | 6 |
| 0111 | 7 | 7 |
| 1000 | 8 | -8 |
| 1001 | 9 | -7 |
| 1010 | 10 | -6 |
| 1011 | 11 | -5 |
| 1100 | 12 | -4 |
| 1101 | 13 | -3 |
| 1110 | 14 | -2 |
| 1111 | 15 | -1 |

Exercise: Numeric ranges

For 16 bit signed numbers (w=16), write the greatest positive value and the most negative value, in hex and decimal. What does -1 look like?

- Greatest positive = 0x7FFF = 32767
- Least negative = 0x8000 = -32768
- Negative 1 = 0xFFFFF

Do the same for 32 bits.

Ranges for Different Word Sizes

| | W | | | |
|---------------------|------|---------|----------------|----------------------------|
| | 8 | 16 | 32 | 64 |
| Unsigned Max | 255 | 65,535 | 4,294,967,295 | 18,446,744,073,709,551,615 |
| Signed Max | 127 | 32,767 | 2,147,483,647 | 9,223,372,036,854,775,807 |
| Signed Min | -128 | -32,768 | -2,147,483,648 | -9,223,372,036,854,775,808 |

Casting Signed to Unsigned

C allows conversions from signed to unsigned

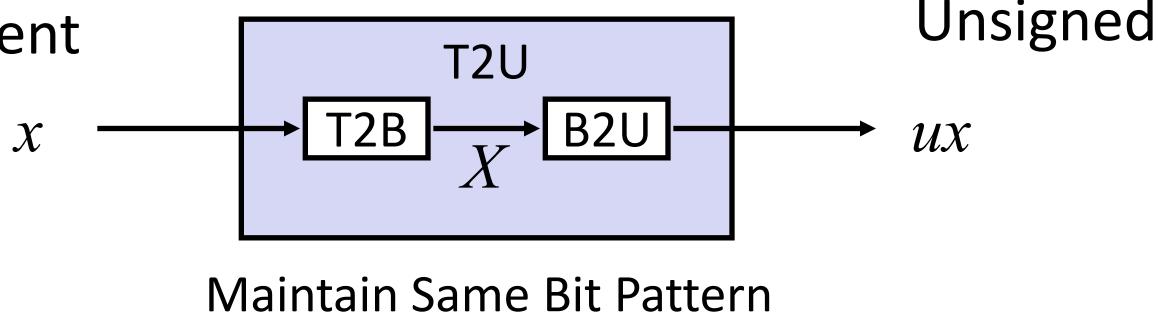
```
short int          x = 15213;
unsigned short int ux = (unsigned short) x;
short int          y = -15213;
unsigned short int uy = (unsigned short) y;
```

Resulting Value

- No change in bit representation
- Non-negative values unchanged
 - $ux = 15213$
- Negative values change into (large) positive values
 - $uy = 50323$
 - Why? MSB treated as large positive number rather than large negative one.

Relation between Signed & Unsigned

Two's Complement



$w-1 \quad \dots \quad 0$

$ux \quad \boxed{+++} \quad \dots \quad \boxed{+++}$

$x \quad \boxed{-++} \quad \dots \quad \boxed{+++}$



Large negative weight

becomes

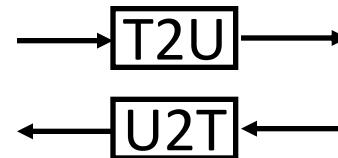
Large positive weight

Mapping Signed / Unsigned

| Bits |
|------|
| 0000 |
| 0001 |
| 0010 |
| 0011 |
| 0100 |
| 0101 |
| 0110 |
| 0111 |
| 1000 |
| 1001 |
| 1010 |
| 1011 |
| 1100 |
| 1101 |
| 1110 |
| 1111 |

| Signed |
|--------|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| -8 |
| -7 |
| -6 |
| -5 |
| -4 |
| -3 |
| -2 |
| -1 |

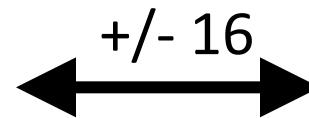
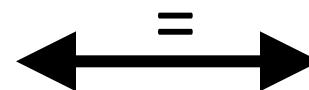
| Unsigned |
|----------|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |
| 13 |
| 14 |
| 15 |



Mapping Signed / Unsigned

| Bits |
|------|
| 0000 |
| 0001 |
| 0010 |
| 0011 |
| 0100 |
| 0101 |
| 0110 |
| 0111 |
| 1000 |
| 1001 |
| 1010 |
| 1011 |
| 1100 |
| 1101 |
| 1110 |
| 1111 |

| Signed |
|--------|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| -8 |
| -7 |
| -6 |
| -5 |
| -4 |
| -3 |
| -2 |
| -1 |



| Unsigned |
|----------|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |
| 13 |
| 14 |
| 15 |

Signed vs. unsigned example

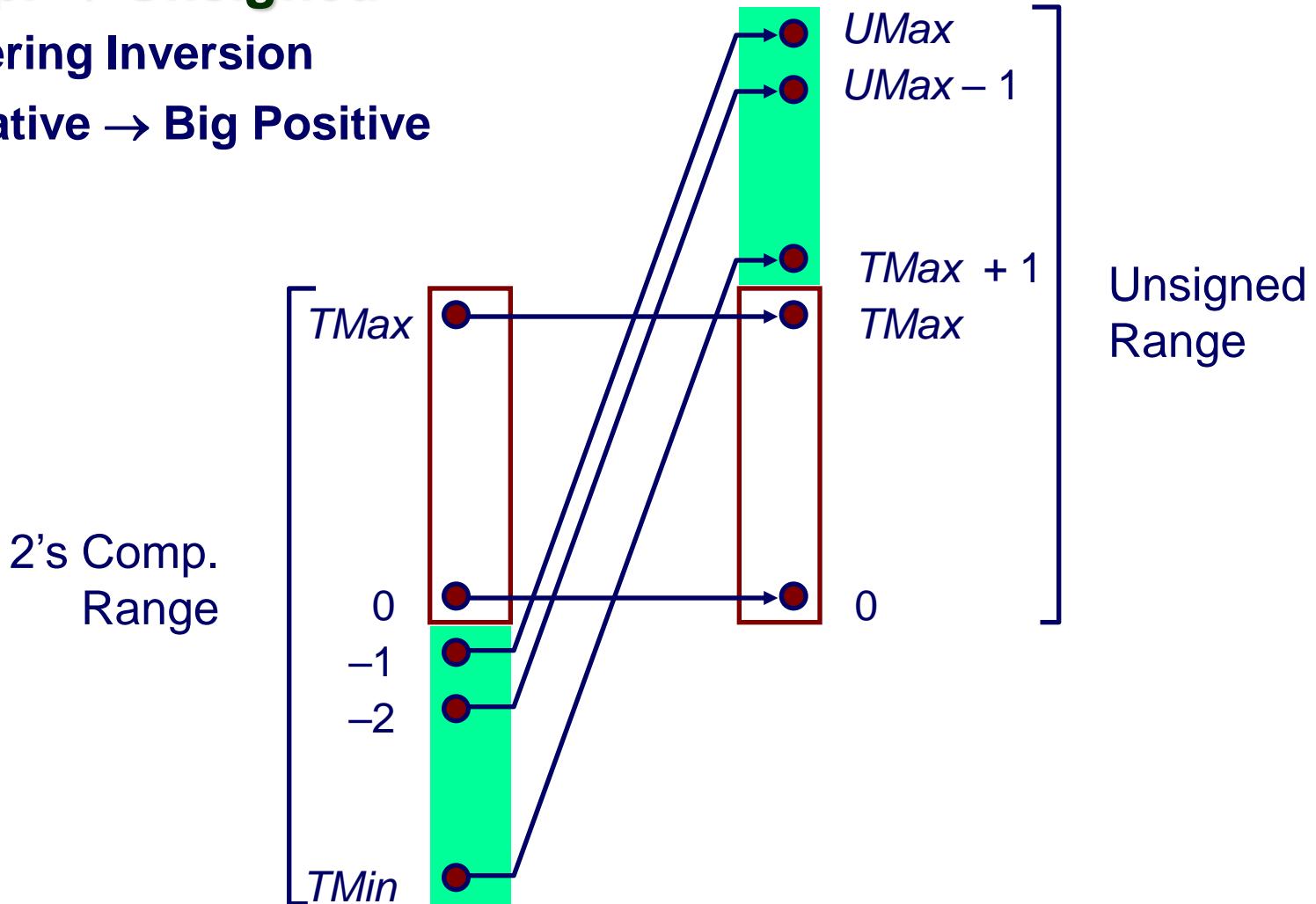
x = 11000100 10010011

| Weight | x unsigned | x signed | | |
|------------|--------------|----------|---------------|--------|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 1 | 2 |
| 4 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 16 | 1 | 16 | 1 | 16 |
| 32 | 0 | 0 | 0 | 0 |
| 64 | 0 | 0 | 0 | 0 |
| 128 | 1 | 128 | 1 | 128 |
| 256 | 0 | 0 | 0 | 0 |
| 512 | 0 | 0 | 0 | 0 |
| 1024 | 1 | 1024 | 1 | 1024 |
| 2048 | 0 | 0 | 0 | 0 |
| 4096 | 0 | 0 | 0 | 0 |
| 8192 | 0 | 0 | 0 | 0 |
| 16384 | 1 | 16384 | 1 | 16384 |
| +32768 | 1 | 32768 | 1 | -32768 |
| Sum | 50323 | | -15213 | |

Conversion visualized

2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive



Example

```
#include <stdio.h>
main()
{
    int i = 0xFFFFFFFF;
    unsigned int j = 0xFFFFFFFF;

    printf("Signed: 0x%x is %d , Unsigned: 0x%x is %u\n",i,i,j,j);

    i=0x80000000;
    j=0x80000000;
    printf("Signed: 0x%x is %d , Unsigned: 0x%x is %u\n",i,i,j,j);

    i=0x7FFFFFFF;
    j=0x7FFFFFFF;
    printf("Signed: 0x%x is %d , Unsigned: 0x%x is %u\n",i,i,j,j);

    short int x = 15213;
    short int y = -15213;

    unsigned short int ux = (unsigned short int) x;
    unsigned short int uy = (unsigned short int) y;
    printf("x:%d y:%d ux:%u uy:%u\n", x,y,ux,uy);
}

Output:
Signed: 0xffffffff is -1 , Unsigned: 0xffffffff is 4294967295

Signed: 0x80000000 is -2147483648 ,
Unsigned: 0x80000000 is 2147483648

Signed: 0x7fffffff is 2147483647 , Unsigned:
0x7fffffff is 2147483647

x:15213 y:-15213 ux:15213 uy:50323
```

Signed vs. Unsigned in C

Constants

- By default are considered to be signed integers
- Unsigned if have “U” as suffix
0U, 4294967259U

Casting

- Explicit casting between signed & unsigned

```
int tx, ty;  
unsigned ux, uy;  
tx = (int) ux;  
uy = (unsigned) ty;
```
- Implicit casting also occurs via assignments and procedure calls

```
tx = ux;  
uy = ty
```

Casting Surprises

Expression Evaluation

- Mixing unsigned and signed in an expression, signed values implicitly cast to unsigned
- Including comparison operations `<`, `>`, `==`, `<=`, `>=`
- Examples for $W = 32$ ($TMIN = -2,147,483,648$, $TMAX = 2,147,483,647$)

| Constant ₁ | Constant ₂ | Relation | Evaluation |
|-----------------------|-----------------------|----------|------------|
| 0 | 0U | == | unsigned |
| -1 | 0 | < | signed |
| -1 | 0U | > | unsigned |
| 2147483647 | -2147483648 | > | signed |
| 2147483647U | -2147483648 | < | unsigned |
| -1 | -2 | > | signed |
| (unsigned) -1 | -2 | > | unsigned |
| 2147483647 | 2147483648U | < | unsigned |
| 2147483647 | (int) 2147483648U | > | signed |

Errors from mixing signed and unsigned

Easy to make mistakes

```
unsigned int i;  
int a[_CNT];  
for (i = CNT-2; i >= 0; i--)  
    a[i] += a[i+1];
```

Is this ever false?

Can be very subtle. (Implicit casting of signed to unsigned)

```
#define DELTA sizeof(int)  
int i;  
for (i = CNT; i-DELTA >= 0; i-= DELTA)  
    . . .
```

Is this ever false?

Counting Down with Unsigned

One potential fix...

```
unsigned i;  
for (i = cnt-2; i < cnt; i--)  
    a[i] += a[i+1];
```



Subtraction at 0 yields large positive number and exits loop

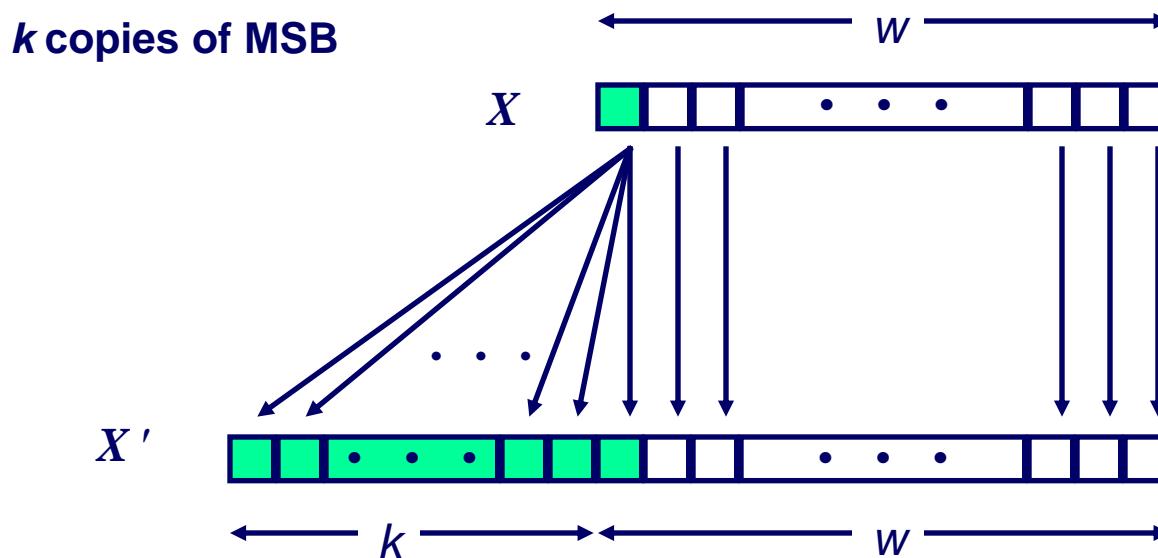
Casting with different integer sizes

Task:

- Given w -bit signed integer x
- Convert it to $w+k$ -bit integer with same value

Rule:

- Make k copies of sign bit:
- $X' = x_{w-1}, \dots, x_{w-1}, x_{w-1}, x_{w-2}, \dots, x_0$



Sign Extension Example

```
short int x = 15213;
int ix = (int) x;
short int y = -15213;
int iy = (int) y;
```

| | Decimal | Hex | | | | Binary | | | |
|----|---------|-----|----|----|----|----------|----------|----------|----------|
| x | 15213 | | 3B | 6D | | | 00111011 | 01101101 | |
| ix | 15213 | 00 | 00 | 3B | 6D | 00000000 | 00000000 | 00111011 | 01101101 |
| y | -15213 | | | C4 | 93 | | | 11000100 | 10010011 |
| iy | -15213 | FF | FF | C4 | 93 | 11111111 | 11111111 | 11000100 | 10010011 |

Converting from smaller to larger integer data type
C automatically performs sign extension

Sign Extension Exercise

Calculate the hex value of -5 for w=4

Calculate the hex value of -5 for w=8

Calculate the hex value of -5 for w=16

Sign extension mayhem

Implicit casting and sign extension leads to errors

```
#include <stdio.h>
main() {
    char c=128;
    unsigned int uc;
    uc = (unsigned int) c;
    printf("%x %u\n",uc, uc);
}
Prints 0xffffffff80 4294967168.
```

Sign extension mayhem

Spot the security bug

```
int sprintf(char *str, size_t size, const char *format, ...);  
  
char dst[257];  
char len;  
len=get_len_field();           /* read 8-bit length field */  
sprintf(dst, len, "%s", src); /* sprintf (size_t) len bytes */
```

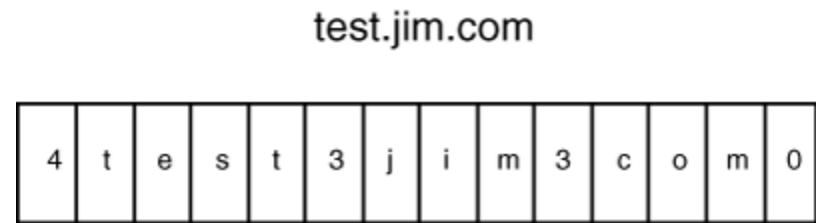
- **get_len_field reads 8-bit integer from network**
- **Can store field in a byte, but unfortunately a signed byte is used**
- **Any length > 128 converts to a negative number**
- **Casting from signed char to unsigned int sign extends!**
- **Input can overflow dst array**

Sign extension mayhem

DNS parser vulnerability

- Format being read: byte length followed by substring

```
char *idx;
int count;
char nameStr[MAX_LEN]; //256
...
memset(nameStr, '\0', sizeof(nameStr));
...
idx = (char *) (pkt + rr_offset);
count = (char)*idx;
while (count) {
    (char *)idx++;
    strncat(nameStr, (char *)idx, count);
    idx += count;
    count = (char)*idx;
    strncat(nameStr, ".", sizeof(nameStr) - strlen(nameStr));
}
nameStr[strlen(nameStr)-1] = '\0';
```



No length check to keep from overflowing nameStr

Count = 128? Negative count that is sign extended in strncat

Type mismatches and security

Comparison vulnerabilities

- Recall mixing casts signed to unsigned
- In `get_int`, what is returned when data is “-1”
- Will `n < 0` ever be true in either `get_int` or `main`?

```
int get_int(char *data) {
    unsigned int n = atoi(data);
    if(n < 0 || n > 1024)
        return -1;
    return n;
}
int main(int argc, char **argv) {
    unsigned long n;
    char buf[1024];
    if(argc < 2)
        exit(0);
    n = get_int(argv[1]);
    if(n < 0){
        fprintf(stderr, "illegal length specified\n");
        exit(-1);
    }
    memset(buf, 'A', n);
    return 0;
}
```

Passing -1 results in `get_int` returning -1 and a large `memset`

Type mismatches and security

2002 FreeBSD getpeername() bug

- Internal code implementing copy of hostname into user buffer used signed int (See B&O Ch. 2 Aside)
- `memcpy` call uses unsigned length
- What if adversary gives a length of “-1” for his buffer size?

```
#define KSIZE 1024
char kbuf[KSIZE]
void *memcpy(void *dest, void *src, size_t n);

int copy_from_kernel(void *user_dest, int maxlen) {
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

- (`KSIZE < -1`) is false, so `len = -1`
- `memcpy` casts -1 to $2^{32}-1$
- Unauthorized kernel memory copied out

Type mismatches and security

Truncation vulnerabilities

- What happens if `userstr` is 65,536 bytes long?

```
unsigned short int f;
char mybuf[1024];
char *userstr=getuserstr();

f=strlen(userstr);
if (f >= sizeof(mybuf))
    die("string too long!");
strcpy(mybuf, userstr);
```

- `strlen` returns `int`, but output truncated to 0
- `strcpy` overruns `mybuf` with entire `userstr` input

Move to 05Arithmetc

Pointer arithmetic

Arithmetic based on size of the type being pointed to

- Incrementing an (int *) adds 4 to pointer
- Incrementing a (char *) adds 1 to pointer

Pointer arithmetic exercise

Consider the following declaration on

```
char* cp=0x100;  
int* ip=0x200;  
float* fp=0x300;  
double* dp=0x400;  
int i=0x500;
```

What are the hexadecimal values of each after execution of these commands?

```
cp++;  
ip++;  
fp++;  
dp++;  
i++;
```

| C Data Type | Typical 32-bit | x86-64 |
|-------------|----------------|--------|
| char | 1 | 1 |
| short | 2 | 2 |
| int | 4 | 4 |
| long | 4 | 8 |
| float | 4 | 4 |
| double | 8 | 8 |
| pointer | 4 | 8 |

Extras

Type mismatches and security

Signed comparison vulnerability example

```
int read_user_data(int sockfd) {
    int length, sockfd, n;
    char buffer[1024];
    length = get_user_length(sockfd);
    if(length > 1024){
        error("illegal input, not enough room in buffer\n");
        return -1;
    }
    if(read(sockfd, buffer, length) < 0){
        error("read: %m");
        return -1;
    }
    return 0;
}
```

- `get_user_length` returns an unsigned 32-bit integer
- Since `length` is signed, what happens on a `length > 231`?
- `length` test passes since `length` negative
- `read` turns `length` into huge positive integer

Pointers and arrays

Arrays

- Stored contiguously in one block of memory
- Index specifies offset from start of array in memory

```
int a[20];
```

- “a” used alone is a pointer containing address of the start of the integer array
- Elements can be accessed using index or via pointer increment and decrement
 - Pointer increments and decrements based on type of array

Example

```
#include <stdio.h>
main()
{
    char* str="abcdefg\n";
    char* x;
    x = str;
    printf("str[0]: %c str[1]: %c str[2]: %c str[3]: %c\n",
           str[0],str[1],str[2],str[3]);

    printf("x: %x *x: %c\n",x,*x);  x++;
    printf("x: %x *x: %c\n",x,*x);  x++;
    printf("x: %x *x: %c\n",x,*x);  x++;
    printf("x: %x *x: %c\n",x,*x);

    int numbers[10], *num, i;
    for (i=0; i < 10; i++)    numbers[i]=i;
    num=(int *) numbers;

    printf("num: %x *num: %d\n",num,*num);  num++;
    printf("num: %x *num: %d\n",num,*num);  num++;
    printf("num: %x *num: %d\n",num,*num);  num++;
    printf("num: %x *num: %d\n",num,*num);

    num=(int *) numbers;
    printf("numbers: %x num: %x &numbers[4]: %x num+4: %x\n",
           numbers, num, &numbers[4],num+4);
    printf("%d %d\n",numbers[4],*(num+4));
}
```

Output:

str[0]: a str[1]: b str[2]: c str[3]: d

x: 8048690 *x: a

x: 8048691 *x: b

x: 8048692 *x: c

x: 8048693 *x: d

num: fffe0498 *num: 0

num: fffe049c *num: 1

num: fffe04a0 *num: 2

num: fffe04a4 *num: 3

numbers: fffe0498 num: fffe0498
&numbers[4]: fffe04a8 num+4: fffe04a8
4 4

Negating with Complement & Increment

For 2's complement, negation can be implemented as the bit-wise complement plus 1

- Claim: $\sim x + 1 == -x$

Complement

- Observation: $\sim x + x == 1111\dots11_2 == -1$

$$\begin{array}{r} x \boxed{1} \boxed{0} \boxed{0} \boxed{1} \boxed{1} \boxed{1} \boxed{0} \\ + \quad \sim x \boxed{0} \boxed{1} \boxed{1} \boxed{0} \boxed{0} \boxed{0} \boxed{1} \\ \hline -1 \boxed{1} \boxed{1} \boxed{1} \boxed{1} \boxed{1} \boxed{1} \boxed{1} \end{array}$$

Increment

- $\sim x + x + (-x + 1) == -1 + (-x + 1)$
- $\sim x + 1 == -x$

Negation with Complementation

4-bit examples

| x | | $\sim x$ | | incr($\sim x$) | |
|------|----|----------|----|------------------|----|
| 0101 | 5 | 1010 | -6 | 1011 | -5 |
| 0111 | 7 | 1000 | -8 | 1001 | -7 |
| 1100 | -4 | 0011 | 3 | 0100 | 4 |
| 0000 | 0 | 1111 | -1 | 0000 | 0 |
| 1000 | -8 | 0111 | 7 | 1000 | -8 |
| | | | | -8 4 2 1 | |

Comp. & Incr. Examples

$x = 15213$

| | Decimal | Hex | Binary | |
|------------|---------|-------|----------|------------------|
| x | 15213 | 3B 6D | 00111011 | 01101101 |
| $\sim x$ | -15214 | C4 92 | 11000100 | 10010010 |
| $\sim x+1$ | -15213 | C4 93 | 11000100 | 1001001 1 |
| y | -15213 | C4 93 | 11000100 | 10010011 |

0

| | Decimal | Hex | Binary | |
|------------|---------|-------|----------|----------|
| 0 | 0 | 00 00 | 00000000 | 00000000 |
| ~ 0 | -1 | FF FF | 11111111 | 11111111 |
| $\sim 0+1$ | 0 | 00 00 | 00000000 | 00000000 |