1. Indicate whether each of the following formulae is a tautology (t), is a contingency (c), or is a contradiction (u for unsatisfiable) by circling the appropriate letter. When the formula is a contingency, give an interpretation that makes it true, and an interpretation that makes it false. *Circle one of* t, c, *or* u.

   (a) *[10 pts] (T) c u :* $A \vee B \vee \neg\, C \vee \neg\, A$

   (b) *[10 pts] t c (U) :* $A \wedge B \wedge C \wedge (B \rightarrow \neg\, A)$

   (c) *[10 pts] t c (U) :* $\neg\, A \wedge \neg\,(A \vee (A \rightarrow A))$

   (d) *[10 pts] t (C) u :* $A \rightarrow \neg\, A$

   A=false; A=true

   (e) *[10 pts] t (C) u :* $(A \vee B) \rightarrow A$

   A=true, B=false; A=false, B=true

2. An industrial spray-and-bake room at a painting company has a number of safeguards to keep employees from injury, while trying to maximize the use of the room.

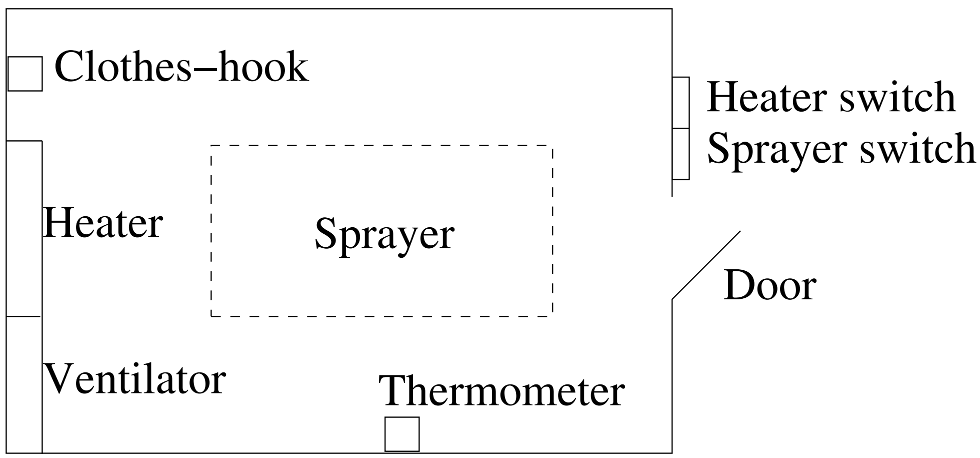

Figure 1: Paint-and-Bake Room, Top View

The room contains three automatically-controlled devices: a paint sprayer, a high-temp heater and a ventilator. It also contains four sensors: a clothes-hook with a switch indicating whether a thermal-protection suit is hanging from it (it is assumed that when the hook is unoccupied the worker is wearing the suit), a door-locked detection switch (the door can be locked only from the outside), a thermometer calibrated to indicate whether the room is too hot to be safe for an unprotected person (too hot to be inside the room but outside the suit), a switch to run the sprayer, and a switch to run the heater.

Operation of the room should obey the following safety and performance conditions:

- The sprayer and heater should never be on at the same time. If both the sprayer and heater switches are on, the sprayer should run.
- The ventilator should be on when the heater switch is on but the sprayer switch is not. It should also be on whenever the door is unlocked, the clothes-hook is occupied, and the temperature is too high.
- The heater should not be on when the door is unlocked unless the clothes-hook is empty or the temperature is low.
- The sprayer and heater should run as often as possible given these conditions, so that machine time is not wasted. The ventilator should off as often as possible, to minimize its use.

As a Z specification we end up with something like this:

$COND ::= on \mid off$
$TEMP ::= hi \mid lo$
$HOOK ::= full \mid empty$
$LOCK ::= locked \mid unlocked$

PaintRoom

$hook : HOOK$
$clotheshook, vent, sprayer : COND$
$temp : TEMP$
$lock : LOCK$

$sprayer = on \Leftrightarrow clotheshook = off$
$clotheshook = on \land sprayer = off \Rightarrow vent = on$
$lock = unlocked \land hook = full \land temp = hi \Rightarrow vent = on$
$lock = unlocked \Rightarrow clotheshook = off \land hook = empty \land temp = lo$

Note that all the state variables can be regarded as boolean literals, with one value being positive and the other negative. This allows us to do propositional proofs in this environment. The constraints become:

$$S \Leftrightarrow \neg C$$
$$C \land \neg S \Rightarrow V$$
$$\neg L \land H \land T \Rightarrow V$$
$$\neg L \Rightarrow \neg C \land \neg H \land \neg T$$

(a) *[25 pts]* Prove: $\neg (S \land C)$

$S \Leftrightarrow \neg C$	[given]
$(\neg S \lor \neg C) \land (C \lor S)$	[def $\Leftrightarrow$]
$\neg (S \land C) \land (C \lor S)$	[de Morgan]
$\neg (S \land C)$	[$\land$-elim]

(b) *[25 pts]* Prove: $\neg L \Rightarrow S$

$\neg L \Rightarrow \neg C \land \neg H \land \neg T$	[given]
$\neg L \Rightarrow \neg C$	[$\land$-elim (1)]
$S \Leftrightarrow \neg C$	[given]
$\neg C \Rightarrow S$	[def $\equiv$, $\land$-elim (2)]
$\neg L \Rightarrow S$	[(1), (2), mp]