1. One of the oldest and most famous propositions in logic is the demonstration of *Modus Ponens* often attributed to Aristotle:

   i) All men are mortal.

   ii) Socrates is a man.

   iii) Socrates is mortal.

   (a) Formalize each statement in first-order logic.

   $$\forall x \bullet man(x) \Rightarrow mortal(x)$$
   $$man(Socrates)$$
   $$mortal(Socrates)$$

   (b) Prove that (iii) is true given (i) and (ii).

$man(x) \Rightarrow mortal(x)$	[1: given (clausal form)]
$man(Socrates) \Rightarrow mortal(Socrates)$	[2: ∀-inst on (1)]
$man(Socrates)$	[3: given]
$mortal(Socrates)$	[mp on (2), (3)]

2. The Drinker's Paradox, popularized Raymond Smullyan, can be stated as "There is someone in the pub such that, if that person is drinking, then everyone in the pub is drinking."

   (a) State the Drinker's Paradox formally.

   $$\exists x \bullet drinking(x) \Rightarrow (\forall y \bullet drinking(y))$$

   (b) Prove that the Drinker's Paradox is a tautology.

   By contradiction:

$\neg \exists x \bullet drinking(x) \Rightarrow (\forall y \bullet drinking(y))$	[1: negation of given]
$\forall x \bullet \neg (drinking(x) \Rightarrow (\forall y \bullet drinking(y)))$	[2: qf on (1)]
$\forall x \bullet drinking(x) \wedge \neg (\forall y \bullet drinking(y))$	[3: def implies, dm on (2)]
$\forall x \bullet \exists y \bullet drinking(x) \wedge \neg drinking(y)$	[4: qf on (3)]
$drinking(x) \wedge \neg drinking(s(x))$	[5: clause + skolem on (4)]
$drinking(k) \wedge \neg drinking(s(k))$	[6: ∀-inst on (5)]
$drinking(s(k)) \wedge \neg drinking(s(s(k)))$	[7: ∀-inst on (5)]
$\square$	[∧-elim on (6), (7); □-intro]