

Yacht: A Yahtzee Variant

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1 Introduction

Yahtzee is a game played with five dice. We will describe a variant of Yahtzee known as Yacht. Yacht is an older game ([https://en.wikipedia.org/wiki/Yacht_\(dice_game\)](https://en.wikipedia.org/wiki/Yacht_(dice_game))) with slightly simpler rules.

Yacht is built around a scoresheet. A scoresheet holds scores for various entries (combinations of dice in a die roll). The scoresheet can be filled in in any order.

$$\begin{aligned} POSN &== 1..5 \\ FACE &== 1..6 \\ ENTRY &::= ks\langle\langle FACE \rangle\rangle \mid full_house \mid four_of_a_kind \mid \\ &\quad little_straight \mid big_straight \mid choice \mid yacht \end{aligned}$$

Scoresheet

$$scoresheet : ENTRY \rightarrow \mathbb{N}$$

A Yacht game starts with an empty scoresheet.

InitYah

Scoresheet

$$scoresheet = \emptyset$$

We will describe a roll of five dice in terms of the values on each of the five top faces.

$$ROLL : \mathbb{P} \text{ seq } FACE$$
$$\forall r : ROLL \bullet \text{dom } r = POSN$$

The sum of the range values of a binary relation will be useful throughout this specification. It should arguably be part of the toolkit—oh well.

[X]

$$sum_range : (X \leftrightarrow \mathbb{Z}) \rightarrow \mathbb{Z}$$
$$sum_range(\emptyset) = 0$$
$$\forall x : X; n : \mathbb{Z}; xns : X \leftrightarrow \mathbb{Z} \bullet$$
$$sum_range(\{x \mapsto n\} \cup xns) = n + sum_range(xns)$$

2 Scoring

We will describe the scores of Yacht entries for now in terms of a scoring function. For a given category, the scoring function assigns a positive score if the roll meets the category description, and assigns a zero score otherwise.

To score a roll as Aces, Twos, Threes etc, we sum the faces with the designated value.

$$SCORE == ENTRY \times ROLL \mapsto \mathbb{N}$$

$$\left| \begin{array}{l} \text{score_ks} : SCORE \\ \hline \text{score_ks} = \{k : FACE; r : ROLL \bullet \\ (ks(k), r) \mapsto \text{sum_range}(r \triangleright \{k\})\} \end{array} \right.$$

To score a roll as a Full House, we give it the sum of all its faces if three of the faces match and the two remaining faces also match; zero otherwise. Yacht may be scored as a Full House.

$$\left| \begin{array}{l} \text{zero} : ENTRY \rightarrow SCORE \\ \hline \forall e : ENTRY \bullet \text{zero}(e) = \{r : ROLL \bullet (e, r) \mapsto 0\} \end{array} \right.$$

$$\left| \begin{array}{l} \text{score_full_house} : SCORE \\ \hline \text{score_full_house} = \text{zero}(\text{full_house}) \oplus \{r : ROLL; x : \mathbb{P} POSN \mid \\ \# x = 3 \wedge \#(r \upharpoonright x) = 1 \wedge \#(r \upharpoonright POSN \setminus x) = 1 \bullet \\ (\text{full_house}, r) \mapsto \text{sum_range}(r)\} \end{array} \right.$$

To score a roll as Four of a Kind, we give it a score equal to the sum of its matching faces if there are at least four matching faces, and zero otherwise. A Yacht may be scored as a Four of a Kind, but receives points only for four of the faces.

$$\left| \begin{array}{l} \text{score_four_of_a_kind} : SCORE \\ \hline \text{score_four_of_a_kind} = \text{zero}(\text{four_of_a_kind}) \oplus \{r : ROLL; x : \mathbb{P} POSN \mid \\ \# x = 4 \wedge \#(r \upharpoonright x) = 1 \bullet \\ (\text{four_of_a_kind}, r) \mapsto \text{sum_range}(x \triangleleft r)\} \end{array} \right.$$

To score a roll as a Little Straight (a better name would be Low Straight), we give it 30 points iff its faces in some order are 1..5.

$$\left| \begin{array}{l} \text{score_little_straight} : SCORE \\ \hline \text{score_little_straight} = \text{zero}(\text{little_straight}) \oplus \{r : ROLL \mid \text{ran}(r) = 1..5 \bullet \\ (\text{little_straight}, r) \mapsto 30\} \end{array} \right.$$

To score a roll as a Big Straight (a better name would be High Straight), we give it 30 points iff its faces in some order are 2..6.

$$\left| \begin{array}{l} \text{score_big_straight} : SCORE \\ \hline \text{score_big_straight} = \text{zero}(\text{big_straight}) \oplus \{r : ROLL \mid \text{ran}(r) = 2..6 \bullet \\ (\text{big_straight}, r) \mapsto 30\} \end{array} \right.$$

To score a roll as Choice, we score it as the sum of its faces.

$$\begin{array}{|l} \hline \textit{score_choice} : \textit{SCORE} \\ \hline \textit{score_choice} = \{r : \textit{ROLL} \bullet (\textit{choice}, r) \mapsto \textit{sum_range}(r)\} \end{array}$$

To score a roll as Yacht, we give it 50 points if all five dice are the same and 0 points otherwise.

$$\begin{array}{|l} \hline \textit{score_yacht} : \textit{SCORE} \\ \hline \textit{score_yacht} = \textit{zero}(\textit{yacht}) \oplus \{r : \textit{ROLL} \mid \# \textit{ran}(r) = 1 \bullet (\textit{yacht}, r) \mapsto 50\} \end{array}$$

The overall score is simply the score obtained by filling in the value of the given roll at the given entry.

$$\begin{array}{|l} \hline \textit{score} : \textit{ENTRY} \times \textit{ROLL} \rightarrow \mathbb{N} \\ \hline \textit{score} = \textit{score_ks} \cup \textit{score_full_house} \cup \textit{score_four_of_a_kind} \cup \\ \textit{score_little_straight} \cup \textit{score_big_straight} \cup \\ \textit{score_choice} \cup \textit{score_yacht} \end{array}$$

3 Play

A turn in Yacht consists of rolling five dice, possibly rerolling once or twice, selecting an entry that matches the final roll, and filling in the scoresheet with the score of that match. An alternative is always to fill in a zero score in any scoresheet entry.

$$\begin{array}{|l} \hline \textit{Dice} \\ \hline \textit{roll} : \textit{ROLL} \\ \textit{count} : \mathbb{N} \\ \hline \textit{count} \leq 2 \end{array}$$

At the start of each turn, an initial roll is made.

$$\begin{array}{|l} \hline \textit{InitTurn} \\ \hline \textit{Dice}' \\ \textit{Scoresheet} \\ \textit{roll}' : \textit{ROLL} \\ \hline \textit{dom scoresheet} \subset \textit{ENTRY} \\ \textit{roll}' = \textit{roll}' \\ \textit{count}' = 2 \end{array}$$

Once the initial roll has been made, the player may select any of the five dice to reroll. This can happen up to twice. Another way to look at it is that the player must twice reroll some subset of the dice, but may choose to reroll none. We will adopt this view, which leads to an easier description.

$$\begin{array}{|l} \hline \textit{Reroll} \\ \hline \Delta \textit{Dice} \\ \textit{Scoresheet} \\ \textit{replace}' : \textit{POSN} \rightarrow \textit{FACE} \\ \hline \textit{count} > 0 \\ \textit{count}' = \textit{count} - 1 \\ \textit{roll}' = \textit{roll} \oplus \textit{replace}' \end{array}$$

When the rerolls are complete, the player selects an empty entry to fill in with the score.

$EndTurn$ $Dice$ $\Delta Scoresheet$ $choice? : ENTRY$
$count = 0$ $choice? \notin \text{dom } scoresheet$ $scoresheet' = scoresheet \cup \{choice? \mapsto \text{score}(choice?, roll)\}$

When all scoresheet entries are filled in, the game is over, and the score is the sum of the values of scoresheet entries.

$EndYah$ $Scoresheet$ $final_score! : \mathbb{N}$
$\text{dom } scoresheet = ENTRY$ $final_score! = \text{sum_range}(scoresheet)$

The game is played from beginning to end.

$$YahGame == InitYah' \circ (InitTurn \circ (Reroll \vee EndTurn) \vee EndYah$$