1. Indicate whether each of the following formulae is a tautology (t), is a contingency (c), or is a contradiction (u for unsatisfiable) by circling the appropriate letter. When the formula is a contingency, give an interpretation that makes it true, and an interpretation that makes it false. Circle one of t, c, or u.

(a) [4 pts] (T) c u : $A \lor B \lor \neg C \lor \neg A$

(b) [4 pts] t c (U) : $A \land B \land C \land (B \rightarrow \neg A)$

(c) [4 pts] t c (U) : $\neg A \land \neg(A \lor (A \rightarrow A))$

(d) [4 pts] t (C) u : $A \rightarrow \neg A$

(e) [4 pts] t (C) u : $(A \lor B) \rightarrow A$

A=false; A=true

A=true, B=false; A=false, B=true
2. An industrial spray-and-bake room at a painting company has a number of safeguards to keep employees from injury, while trying to maximize the use of the room.

The room contains two automatically-controlled devices: a paint sprayer with a high-temperature heater that actually do the paint-and-bake ($D_C$, cycle is running when true) and a ventilator ($D_V$, on when true). It also contains four sensors: a clothes-hook with a switch indicating whether a thermal-protection suit is hanging from it ($S_P$, true when hook occupied—it is assumed that when the hook is unoccupied the worker is wearing the suit), a door-locked detection switch ($S_L$, true when locked—the door can be locked only from the outside), a thermometer calibrated to indicate whether the room is too hot to be safe for an unprotected person ($S_T$, true when too hot to be inside the room but outside the suit), a switch to operate the room ($S_C$, true when the room is supposed to be running).

For each of the device outputs $D$, write a propositional formula in terms of the sensor inputs $S$ that describes their operation. The formula should describe when the device output should be true, satisfying the following safety and performance conditions:

- The heater/spray cycle should be running when the operation switch is on. However, the cycle should not run when the door is unlocked; neither should it run when the clothes-hook is occupied and the temperature is too high.

- The ventilator should be on when the operation switch is on. It should also be on whenever the door is unlocked, the clothes-hook is occupied, and the temperature is too high.

$D_C$ should be true as often as possible given these conditions, so that machine time is not wasted. $D_V$ should be false as often as possible, to minimize use of the ventilator.
(a) [25 pts] Write a formula for $D_C$.

\[ D_C \equiv S_C \land S_L \land \neg(S_P \land S_T) \]

(b) [25 pts] Write a formula for $D_V$.

\[ D_V \equiv S_C \lor (\neg S_L \land S_P \land S_T) \]

(c) [30 pts] Given your formula for $D_V$ above, show that when the ventilator is on, if the cycle switch is off then the temperature is too high. Use a deductive proof.

\[
D_V \rightarrow (S_C \rightarrow S_L)
\]

1. $D_V$ [P]
2. $S_C \lor (\neg S_L \land S_P \land \neg S_T)$ [1, def. $D_V$]
3. $\neg S_C \rightarrow (\neg S_L \land S_P \land \neg S_T)$ [2, def. $\rightarrow$]
4. $\neg S_C$ [P]
5. $\neg S_L \land S_P \land S_T$ [4, 3, MP]
6. $S_T$ [5, Simp]
7. $\neg S_C \rightarrow S_T$ [4–6, CP]
\[ \therefore \] [1–7, CP]